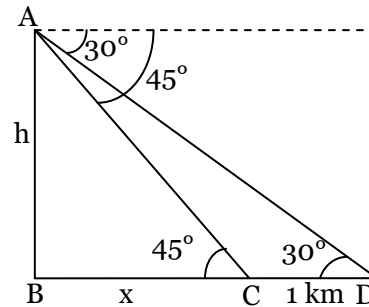


Some Applications of Trigonometry

86. From the top of a hill, the angles of depression of two consecutive kilometers stones east are found to be 30° and 45° . Find the height of the hill?

2014/2015 [3 marks]

Let h be the height of hill AB and C and D is the positions of two consecutive kilometers stones.



Also, let $BC = x$

From $\text{rt.}\triangle ABC$,

$$\begin{aligned}\Rightarrow \tan 45^\circ &= \frac{AB}{BC} \\ \Rightarrow 1 &= \frac{h}{x} \\ \Rightarrow x &= h \dots \dots \dots (1)\end{aligned}$$

From $\text{rt.}\triangle ABD$

$$\begin{aligned}\Rightarrow \tan 30^\circ &= \frac{AB}{BD} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{h}{x+1} \\ \Rightarrow \sqrt{3} \times h &= x+1 \\ \Rightarrow h\sqrt{3} &= h+1 && \text{[From (1)]} \\ \Rightarrow h(\sqrt{3}-1) &= 1 \\ \Rightarrow h &= \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2} \text{ km}\end{aligned}$$

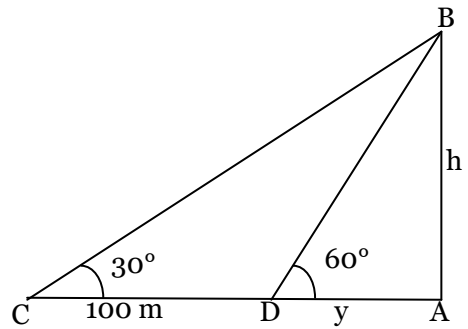
$$\therefore \text{Height of hill} = \frac{(1+\sqrt{3})}{2} \text{ km}$$

87. A man observes the angle of elevation of a bird to be 30° . He then walks 100m towards the bird which is stationary and finds that the angle of elevation is 60° . Find the height at which the bird is sitting?

2014/2015 [3 Marks]

Let B be the position of the bird which is at a height h from the ground level.
Let C and D be the positions of the man.





From rt. $\triangle ABD$,

$$\begin{aligned}\Rightarrow \tan 60^\circ &= \frac{AB}{AD} \\ \Rightarrow \sqrt{3} &= \frac{h}{y} \\ \Rightarrow h &= y\sqrt{3} \dots \dots \dots (1)\end{aligned}$$

From rt. $\triangle ABC$,

$$\begin{aligned}\Rightarrow \tan 30^\circ &= \frac{AB}{AC} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{h}{y+100} \\ \Rightarrow y+100 &= h\sqrt{3} \\ \Rightarrow y &= h\sqrt{3} - 100\end{aligned}$$

From eq. (1),

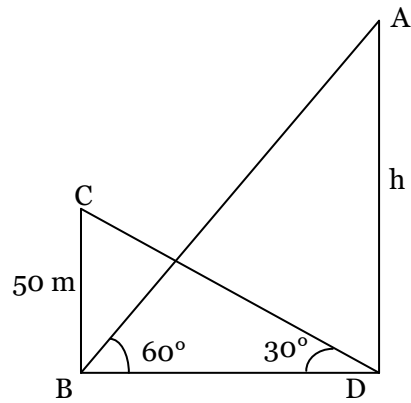
$$\begin{aligned}\Rightarrow h &= (h\sqrt{3} - 100)\sqrt{3} \\ \Rightarrow h &= 3h - 100\sqrt{3} \\ \Rightarrow 2h &= 100\sqrt{3} \\ \Rightarrow h &= 50\sqrt{3} \text{ m} \\ \Rightarrow h &= 50 \times 1.732 \text{ m} \\ \Rightarrow h &= 86.6 \text{ m}\end{aligned}$$

So, bird is sitting at a height of 86.6 m.

88. The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30° . If the tower is 50 m high, find the height of the hill?

2012/2015 [3 Marks]

Let AB be the hill and CD be the tower.



We have:

$$\Rightarrow \angle ADB = 60^\circ \text{ and } \angle CBD = 30^\circ$$

From $\triangle BCD$, we get

$$\begin{aligned} \Rightarrow \frac{50}{BD} &= \tan 30^\circ \\ \Rightarrow \frac{50}{BD} &= \frac{1}{\sqrt{3}} \\ \Rightarrow BD &= 50\sqrt{3} \text{ m.} \end{aligned}$$

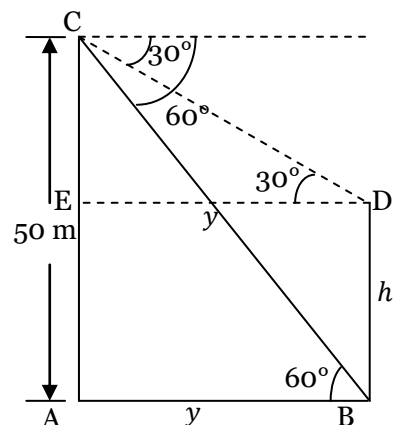
From $\triangle ABD$, we get

$$\begin{aligned} \Rightarrow \frac{AB}{BD} &= \tan 60^\circ \\ \Rightarrow \frac{h}{50\sqrt{3}} &= \sqrt{3} \\ \Rightarrow h &= 150 \text{ m.} \end{aligned}$$

89. There are two temples, one on each bank of a river, just opposite to each other. One temple is 50m high. From top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple?

2014/2015 [4 Marks]

Let AB be the width of the river, AC and BD are the temples of heights 50 m and h m respectively.



From rt. CED, $\triangle$

$$\begin{aligned} \Rightarrow \tan D &= \frac{CE}{ED} \\ \Rightarrow \tan 30^\circ &= \frac{50-h}{y} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{50-h}{y} \\ \Rightarrow y &= \sqrt{3}(50-h) \end{aligned} \quad \dots\dots\dots(1)$$

From rt. ABC, $\triangle$

$$\begin{aligned} \Rightarrow \tan B &= \frac{AC}{AB} \\ \Rightarrow \tan 60^\circ &= \frac{50}{y} \\ \Rightarrow \tan 60^\circ &= \frac{50}{y} \\ \Rightarrow \sqrt{3} &= \frac{50}{y} \\ \Rightarrow y &= \frac{50}{\sqrt{3}} m. \end{aligned}$$

So, from equation (1),

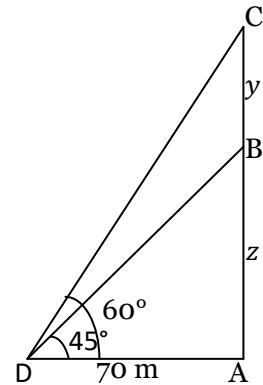
$$\begin{aligned} \Rightarrow \frac{50}{\sqrt{3}} &= \sqrt{3}(50-h) \\ \Rightarrow 50 &= 3(50-h) \\ \Rightarrow 50 &= 150-3h \\ \Rightarrow 3h &= 100 \\ \Rightarrow h &= \frac{100}{3} m. \end{aligned}$$

Hence, width of river = $\frac{50}{\sqrt{3}}$ m and height of the other temple = $\frac{100}{3}$ m.

90. A tower stands on a horizontal plane and is surrounded by a flagstaff. At a point on the plane, 70 meters away from the tower, an observer notices that the angles of elevation of the top and bottom of the flagstaff are respectively 60° and 45° . Find the height of the flagstaff and that of the tower?

2014/2015 [4 Marks]

Let BC be the flagstaff of height y m fixed on a tower AB of height x m.



From rt. $\triangle ABD$,

$$\begin{aligned}\Rightarrow \tan 45^\circ &= \frac{AB}{AD} \\ \Rightarrow \tan 45^\circ &= \frac{x}{70} \\ \Rightarrow 1 &= \frac{x}{70} \\ \Rightarrow x &= 70 \text{ m}\end{aligned}$$

From rt. $\triangle ACD$,

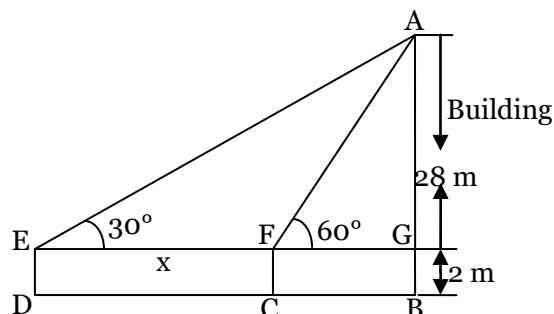
$$\begin{aligned}\Rightarrow \tan 60^\circ &= \frac{AC}{AD} \\ \Rightarrow \tan 60^\circ &= \frac{x+y}{70} \\ \Rightarrow \sqrt{3} &= \frac{70+y}{70} \\ \Rightarrow 70+y &= 70\sqrt{3} \\ \Rightarrow y &= 70(\sqrt{3}-1) \text{ m} \\ \Rightarrow y &= 70 \times 0.73 \text{ m} \\ \Rightarrow y &= 51.1 \text{ m}\end{aligned}$$

Thus, height of flagstaff is 51.1 m and height of tower is 70 m.

91. A boy 2 m tall is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building?

2011/2015 [4 Marks]

Let AB be the building and ED be the boy. E and F are two positions of the eyes of the boy.



Let $EF = x$
In rt. $\triangle AG$,

$$\begin{aligned}\Rightarrow \tan 60^\circ &= \frac{AG}{FG} \\ \Rightarrow \frac{28 - 2}{FG} &= \sqrt{3} \\ \Rightarrow FG &= \frac{28}{\sqrt{3}} = \frac{28\sqrt{3}}{3} m\end{aligned}$$

In rt. $\triangle AEG$,

$$\begin{aligned}\Rightarrow \tan 30^\circ &= \frac{AG}{EG} \\ \Rightarrow \frac{28}{x + \frac{28\sqrt{3}}{3}} &= \frac{1}{\sqrt{3}} \quad \quad \quad [\text{From (1)}]\end{aligned}$$

$$\begin{aligned}\Rightarrow 28\sqrt{3} &= x + \frac{28\sqrt{3}}{3} \\ \Rightarrow x &= 28\sqrt{3} - \frac{28\sqrt{3}}{3} = \frac{56\sqrt{3}}{3} m\end{aligned}$$

Thus, the distance the boy walked towards the building is $\frac{56\sqrt{3}}{3} m$.